

# Quantitative Aptitude Formulas for Placement Tests

A formula sheet for the quantitative section of placement aptitude tests, from percentages and interest to time and work, speed, permutations, probability and number system, with a worked example for each topic.

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## Quantitative Aptitude Formulas for Placement Tests

The quantitative sections of placement aptitude tests draw on a familiar set of topics, and many of their questions reduce to a few formulas. This sheet collects those formulas by topic, with one worked example each so you can see them applied.

Formulas save time only when you know why they work. Try to derive each one once from first principles: if you forget it under time pressure, you can still reason your way to the answer. Throughout,  $\times$  means multiply,  $/$  means divide and  $^$  means "to the power of", so  $(1 + R/100)^n$  is  $(1 + R/100)$  multiplied by itself  $n$  times.

## Percentages

| To find                    | Formula                                           |
|----------------------------|---------------------------------------------------|
| $x\%$ of $N$               | $N \times x/100$                                  |
| $A$ as a percentage of $B$ | $A/B \times 100$                                  |
| Percentage change          | $(\text{new} - \text{old})/\text{old} \times 100$ |

| To find                                          | Formula                                                        |
|--------------------------------------------------|----------------------------------------------------------------|
| N increased by x%                                | $N \times (100 + x)/100$                                       |
| N decreased by x%                                | $N \times (100 - x)/100$                                       |
| Net change after successive changes of a% and b% | $a + b + a \times b/100$ (use a negative value for a decrease) |
| A is x% more than B, so B is less than A by      | $x/(100 + x) \times 100$ %                                     |
| A is x% less than B, so B is more than A by      | $x/(100 - x) \times 100$ %                                     |
| Value after n years of r% growth a year          | $P \times (1 + r/100)^n$                                       |
| Value after n years of r% depreciation a year    | $P \times (1 - r/100)^n$                                       |

The same idea covers price and consumption: if a price rises by x%, consumption must fall by  $x/(100 + x) \times 100$  % to keep spending unchanged. If the price falls by x%, consumption can rise by  $x/(100 - x) \times 100$  %.

Knowing common fractions as percentages speeds up most of these calculations (values rounded to two decimal places):

| Fraction | Percent | Fraction | Percent |
|----------|---------|----------|---------|
| 1/2      | 50%     | 1/8      | 12.5%   |
| 1/3      | 33.33%  | 1/9      | 11.11%  |
| 1/4      | 25%     | 1/10     | 10%     |
| 1/5      | 20%     | 1/11     | 9.09%   |
| 1/6      | 16.67%  | 1/12     | 8.33%   |
| 1/7      | 14.29%  | 1/16     | 6.25%   |

### Percentage points are not percent

If a rate rises from 10% to 12%, it has risen by 2 percentage points, which is a 20% increase. Check which one the question asks for.

**Worked example.** The price of sugar rises by 25%. By what percentage must a family reduce its consumption so that its spending stays the same?

Reduction =  $\frac{25}{(100 + 25)} \times 100 = \frac{25}{125} \times 100 = \mathbf{20\%}$ . Check: at a price of 100 per kg, 1 kg costs 100. At the new price of 125 per kg, 0.8 kg costs 100, which is 20% less sugar.

## Profit, loss and discount

CP is the cost price, SP the selling price and MP the marked (list) price. Profit and loss percentages are calculated on CP; discount percentages are calculated on MP.

| To find                                                       | Formula                                 |
|---------------------------------------------------------------|-----------------------------------------|
| Profit                                                        | $SP - CP$                               |
| Loss                                                          | $CP - SP$                               |
| Profit %                                                      | $\text{Profit}/CP \times 100$           |
| Loss %                                                        | $\text{Loss}/CP \times 100$             |
| SP, given CP and profit %                                     | $CP \times (100 + \text{Profit\%})/100$ |
| SP, given CP and loss %                                       | $CP \times (100 - \text{Loss\%})/100$   |
| CP, given SP and profit %                                     | $SP \times 100/(100 + \text{Profit\%})$ |
| CP, given SP and loss %                                       | $SP \times 100/(100 - \text{Loss\%})$   |
| Discount                                                      | $MP - SP$                               |
| Discount %                                                    | $\text{Discount}/MP \times 100$         |
| SP after a discount of d%                                     | $MP \times (100 - d)/100$               |
| Single discount equal to successive discounts of d1% and d2%  | $d1 + d2 - d1 \times d2/100$            |
| Profit % when marked up m% above CP and sold at a d% discount | $m - d - m \times d/100$                |

| To find                                                            | Formula                           |
|--------------------------------------------------------------------|-----------------------------------|
| MP needed for a profit of p% after a discount of d%                | $CP \times (100 + p)/(100 - d)$   |
| Two items sold at the same SP, one at x% profit and one at x% loss | overall loss of $x^2/100$ %       |
| Dealer sells at cost price but gives W' grams for every W grams    | gain % = $(W - W')/W' \times 100$ |

**Worked example.** A shopkeeper marks goods 40% above cost price and offers a 10% discount. What is the profit percentage?

Using  $m - d - m \times d/100$ :  $40 - 10 - (40 \times 10)/100 = 40 - 10 - 4 = \mathbf{26\%}$ . Check with CP = 100: MP = 140, SP =  $140 \times 0.9 = 126$ , so the profit is 26 on a cost of 100.

## Simple and compound interest

P is the principal, R the rate per cent per year, T or n the time in years, and A the amount (principal plus interest).

| To find                             | Formula                                                        |
|-------------------------------------|----------------------------------------------------------------|
| Simple interest                     | $SI = P \times R \times T/100$                                 |
| Amount with simple interest         | $A = P + SI$                                                   |
| Amount, compounded yearly           | $A = P \times (1 + R/100)^n$                                   |
| Amount, compounded half-yearly      | $A = P \times (1 + R/200)^{(2n)}$                              |
| Amount, compounded quarterly        | $A = P \times (1 + R/400)^{(4n)}$                              |
| Compound interest                   | $CI = A - P$                                                   |
| Different rates in successive years | $A = P \times (1 + R_1/100) \times (1 + R_2/100) \times \dots$ |
| CI minus SI for 2 years             | $P \times (R/100)^2$                                           |
| CI minus SI for 3 years             | $P \times (R/100)^2 \times (3 + R/100)$                        |

**To find****Formula**

Rate at which a sum doubles in T years at SI

$$R = 100/T$$

Sum becomes k times in T years at SI

$$R \times T = 100 \times (k - 1)$$

At compound interest, if a sum becomes x times itself in n years, it becomes  $x^2$  times in 2n years and  $x^3$  times in 3n years.

**Worked example.** Find the difference between compound and simple interest on ₹5,000 for 2 years at 8% a year.

Using  $P \times (R/100)^2$ :  $5,000 \times (8/100)^2 = 5,000 \times 0.0064 = \text{₹}32$ . Check: SI =  $5,000 \times 8 \times 2/100 = \text{₹}800$ . The compound amount is  $5,000 \times 1.08^2 = 5,000 \times 1.1664 = \text{₹}5,832$ , so CI = ₹832, and  $832 - 800 = \text{₹}32$ .

If a question does not say how interest is compounded, assume yearly. Convert months to years before substituting (9 months is 0.75 years), and for half-yearly or quarterly compounding change both the rate and the number of periods.

## Ratio, proportion and averages

### Ratio and proportion

**To find****Formula**

Combine  $a:b = m:n$  and  $b:c = p:q$

$$a:b:c = m \times p : n \times p : n \times q$$

Split an amount S in the ratio a:b

$$S \times a/(a + b) \text{ and } S \times b/(a + b)$$

$a:b = c:d$  (a proportion)

$$a \times d = b \times c$$

Fourth proportional to a, b and c

$$b \times c/a$$

Third proportional to a and b

$$b^2/a$$

Mean proportional of a and b

$$\sqrt{(a \times b)}$$

### Averages and mixtures

| To find                                                                             | Formula                                                                   |
|-------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| Average                                                                             | $\frac{\text{sum of values}}{\text{number of values}}$                    |
| Average of the first $n$ natural numbers                                            | $\frac{(n + 1)}{2}$                                                       |
| Average of evenly spaced numbers                                                    | $\frac{(\text{first} + \text{last})}{2}$                                  |
| Combined average of two groups                                                      | $\frac{(n_1 \times a_1 + n_2 \times a_2)}{(n_1 + n_2)}$                   |
| New member joins a group of $n$ and the average rises by $k$                        | $\text{new member} = \frac{\text{old average} \times (n + 1) + k}{n + 1}$ |
| New member joins a group of $n$ and the average falls by $k$                        | $\text{new member} = \frac{\text{old average} \times (n + 1) - k}{n + 1}$ |
| One of $n$ values is replaced; the average changes by                               | $\frac{(\text{new value} - \text{old value})}{n}$                         |
| Alligation: mix items priced $c$ (cheaper) and $d$ (dearer) to get a mean price $m$ | $\text{cheaper} : \text{dearer} = \frac{(d - m)}{(m - c)}$                |

**Worked example.** The average age of 30 students is 15 years. When the teacher's age is included, the average rises by 1 year. How old is the teacher?

New member =  $15 + \frac{(30 + 1) \times 1}{30 + 1} = 46$  years. Check: the students' ages total  $30 \times 15 = 450$ . With the teacher, the total is  $31 \times 16 = 496$ , and  $496 - 450 = 46$ .

## Time and work

If someone completes a job in  $a$  days, they do  $\frac{1}{a}$  of it each day. Add rates, not days.

| To find                                             | Formula                                                              |
|-----------------------------------------------------|----------------------------------------------------------------------|
| One day's work of A, who finishes alone in $a$ days | $\frac{1}{a}$                                                        |
| Time for A and B together                           | $\frac{a \times b}{(a + b)}$                                         |
| Time for A, B and C together                        | $\frac{a \times b \times c}{(a \times b + b \times c + c \times a)}$ |

| To find                                                              | Formula                                                 |
|----------------------------------------------------------------------|---------------------------------------------------------|
| Time for B alone, if A takes a days and A and B together take x days | $a \times x / (a - x)$                                  |
| Same kind of work done by different teams                            | $M1 \times D1 \times H1/W1 = M2 \times D2 \times H2/W2$ |
| A is k times as efficient as B                                       | A takes $1/k$ of B's time                               |

In the team formula, M is the number of people, D the days, H the hours per day and W the amount of work. When people are paid jointly for a job, the payment is split in the ratio of the work each one did.

A faster method than fractions: take the total work as the LCM of the individual times. Each person's daily work is then a whole number of units.

**Worked example.** A can finish a job in 12 days and B in 18 days. They work together for 4 days, then A leaves. How many more days does B need?

Take the total work as  $\text{LCM}(12, 18) = 36$  units. A does  $36/12 = 3$  units a day and B does  $36/18 = 2$ . In 4 days together they complete  $4 \times 5 = 20$  units, leaving 16. B needs  $16/2 = 8$  more days.

## Pipes and cisterns

These are time and work problems where emptying counts as negative work. An inlet that fills a tank in a hours adds  $1/a$  of the tank per hour; an outlet that empties it in b hours removes  $1/b$ .

| Situation                                                       | Time                                                         |
|-----------------------------------------------------------------|--------------------------------------------------------------|
| Two inlets taking a and b hours                                 | $a \times b / (a + b)$ to fill                               |
| Inlet taking a hours, outlet taking b hours, with $b > a$       | $a \times b / (b - a)$ to fill                               |
| Inlet taking a hours, outlet taking b hours, with $b < a$       | $a \times b / (a - b)$ to empty a full tank                  |
| Pipe fills in a hours alone but takes t hours because of a leak | the leak alone empties a full tank in $a \times t / (t - a)$ |

| Situation           | Time                                        |
|---------------------|---------------------------------------------|
| Any number of pipes | add the rates, counting outlets as negative |

**Worked example.** Pipes A and B fill a tank in 20 and 30 minutes, and pipe C empties it in 15 minutes. If all three are opened on an empty tank, how long does it take to fill?

Take the tank as  $\text{LCM}(20, 30, 15) = 60$  units. A adds 3 units a minute, B adds 2 and C removes 4, so the net rate is  $3 + 2 - 4 = 1$  unit a minute. The tank fills in **60 minutes**.

## Speed, distance and time

Here  $x$  and  $y$  are speeds,  $L$  lengths and  $t$  times.

| To find                                                                                 | Formula                                     |
|-----------------------------------------------------------------------------------------|---------------------------------------------|
| Distance                                                                                | $\text{speed} \times \text{time}$           |
| km/h to m/s                                                                             | multiply by $5/18$                          |
| m/s to km/h                                                                             | multiply by $18/5$                          |
| Average speed                                                                           | $\text{total distance} / \text{total time}$ |
| Average speed over two equal distances at $x$ and $y$                                   | $2 \times x \times y / (x + y)$             |
| Relative speed, same direction                                                          | $x - y$                                     |
| Relative speed, opposite directions                                                     | $x + y$                                     |
| Train of length $L$ passes a pole or a standing person                                  | $\text{time} = L / \text{speed}$            |
| Train of length $L$ passes a platform or bridge of length $P$                           | $\text{time} = (L + P) / \text{speed}$      |
| Trains of lengths $L_1$ and $L_2$ pass each other in opposite directions                | $\text{time} = (L_1 + L_2) / (x + y)$       |
| Trains of lengths $L_1$ and $L_2$ pass each other in the same direction ( $x$ faster)   | $\text{time} = (L_1 + L_2) / (x - y)$       |
| Two objects start towards each other and, after meeting, take $t_1$ and $t_2$ to finish | $x : y = \sqrt{t_2} : \sqrt{t_1}$           |

| To find                                                                          | Formula                                            |
|----------------------------------------------------------------------------------|----------------------------------------------------|
| Same distance, speeds in the ratio $a:b$                                         | times in the ratio $b : a$                         |
| At speed $x$ you arrive $t_1$ late; at a faster speed $y$ you arrive $t_2$ early | distance = $x \times y \times (t_1 + t_2)/(y - x)$ |

### Average speed is not the average of the speeds

Going somewhere at 40 km/h and returning at 60 km/h gives an average of 48 km/h, not 50, because you spend longer at the slower speed. Use total distance over total time.

**Worked example.** A 200 m train running at 72 km/h crosses a 250 m platform. How long does it take?

72 km/h =  $72 \times 5/18 = 20$  m/s. The train must cover its own length plus the platform:  $200 + 250 = 450$  m. Time =  $450/20 = \mathbf{22.5 \text{ seconds}}$ .

## Boats and streams

$u$  is the boat's speed in still water and  $v$  the speed of the stream, with  $u > v$ .

| To find                                                                           | Formula                                               |
|-----------------------------------------------------------------------------------|-------------------------------------------------------|
| Downstream speed                                                                  | $u + v$                                               |
| Upstream speed                                                                    | $u - v$                                               |
| Speed in still water                                                              | $(\text{downstream speed} + \text{upstream speed})/2$ |
| Speed of the stream                                                               | $(\text{downstream speed} - \text{upstream speed})/2$ |
| Time for a round trip of distance $D$ each way                                    | $2 \times u \times D/(u^2 - v^2)$                     |
| Going upstream takes $k$ times as long as going downstream over the same distance | $u : v = (k + 1) : (k - 1)$                           |

**Worked example.** A boat covers 24 km downstream in 2 hours and returns in 3 hours. Find its speed in still water and the speed of the stream.

Downstream speed =  $24/2 = 12$  km/h and upstream speed =  $24/3 = 8$  km/h. Speed in still water =  $(12 + 8)/2 = \mathbf{10 \text{ km/h}}$ , and the stream's speed =  $(12 - 8)/2 = \mathbf{2 \text{ km/h}}$ .

## Permutations and combinations

Use permutations when order matters (arrangements, rankings, codes) and combinations when it doesn't (teams, committees, selections).

| To find                                                                                                | Formula                                                                      |
|--------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| Factorial                                                                                              | $n! = n \times (n - 1) \times \dots \times 2 \times 1$ ,<br>and $0! = 1$     |
| Arrangements of $r$ items chosen from $n$                                                              | $nPr = n!/(n - r)!$                                                          |
| Selections of $r$ items from $n$                                                                       | $nCr = n!/(r! \times (n - r)!)$                                              |
| Useful identities                                                                                      | $nCr = nC(n - r)$ , $nPr = nCr \times r!$ ,<br>$nCr + nC(r - 1) = (n + 1)Cr$ |
| Arrangements of $n$ items with $p$ , $q$ and $r$ identical items of three kinds                        | $n!/(p! \times q! \times r!)$                                                |
| $n$ distinct items in a circle                                                                         | $(n - 1)!$                                                                   |
| $n$ distinct items in a circle where clockwise and anticlockwise count as the same (necklace, garland) | $(n - 1)!/2$                                                                 |
| $n$ distinct items in a row with $r$ particular items always together                                  | $(n - r + 1)! \times r!$                                                     |
| Selecting at least one item from $n$ distinct items                                                    | $2^n - 1$                                                                    |
| Handshakes among $n$ people                                                                            | $nC2 = n \times (n - 1)/2$                                                   |
| Diagonals of a polygon with $n$ sides                                                                  | $n \times (n - 3)/2$                                                         |

| To find                                                    | Formula               |
|------------------------------------------------------------|-----------------------|
| n identical items shared among r people, zero allowed      | $(n + r - 1)C(r - 1)$ |
| n identical items shared among r people, at least one each | $(n - 1)C(r - 1)$     |

When a task happens in steps (this and then that), multiply the counts. When there are separate cases (this or that), add them.

**Worked example.** A committee of 3 men and 2 women is chosen from 6 men and 5 women. How many different committees are possible?

Men:  ${}^6C_3 = 6!/(3! \times 3!) = 20$ . Women:  ${}^5C_2 = 5!/(2! \times 3!) = 10$ . Both choices must happen, so multiply:  $20 \times 10 = \mathbf{200}$ .

## Probability

For equally likely outcomes,  $P(E) = \text{favourable outcomes} / \text{total outcomes}$ , which always lies between 0 and 1.

| Rule                                                          | Formula                                                  |
|---------------------------------------------------------------|----------------------------------------------------------|
| Complement                                                    | $P(\text{not } E) = 1 - P(E)$                            |
| At least one                                                  | $1 - P(\text{none})$                                     |
| A or B                                                        | $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ |
| A or B, when they cannot happen together (mutually exclusive) | $P(A) + P(B)$                                            |
| A and B, when one does not affect the other (independent)     | $P(A) \times P(B)$                                       |
| A, given that B has happened                                  | $P(A \text{ and } B) / P(B)$                             |
| Odds in favour of a:b                                         | $P = a / (a + b)$                                        |

Mutually exclusive and independent are different ideas: if two events each have a non-zero chance, they cannot be both.

Sample spaces worth remembering:

- **Coins.**  $n$  coins give  $2^n$  outcomes.
- **Dice.** One die gives 6 outcomes, two dice 36, three dice 216. With two dice, a sum of  $s$  can be made in  $s - 1$  ways for  $s$  from 2 to 7, and  $13 - s$  ways for  $s$  from 7 to 12.
- **Cards.** A standard deck has 52 cards in 4 suits of 13. There are 26 red cards (hearts and diamonds), 26 black (spades and clubs), 12 face cards (jack, queen and king of each suit) and 4 aces.

**Worked example.** Two cards are drawn at random, without replacement, from a standard deck. What is the probability that both are aces?

Ways to choose 2 aces:  $4C2 = 6$ . Ways to choose any 2 cards:  $52C2 = 52 \times 51/2 = 1,326$ . Probability =  $6/1,326 = 1/221$ . Check the other way:  $(4/52) \times (3/51) = 12/2,652 = 1/221$ .

## Number system

### Divisibility rules

Divisible  
by

Rule

|   |                                                                                                                                        |
|---|----------------------------------------------------------------------------------------------------------------------------------------|
| 2 | the last digit is even                                                                                                                 |
| 3 | the sum of the digits is divisible by 3                                                                                                |
| 4 | the number formed by the last two digits is divisible by 4                                                                             |
| 5 | the last digit is 0 or 5                                                                                                               |
| 6 | divisible by both 2 and 3                                                                                                              |
| 7 | double the last digit and subtract it from the number formed by the remaining digits; repeat until small; the result is divisible by 7 |
| 8 | the number formed by the last three digits is divisible by 8                                                                           |
| 9 | the sum of the digits is divisible by 9                                                                                                |

## Divisible

### by Rule

|    |                                                                                           |
|----|-------------------------------------------------------------------------------------------|
| 10 | the last digit is 0                                                                       |
| 11 | the sum of the digits in odd places minus the sum in even places is 0 or a multiple of 11 |
| 12 | divisible by both 3 and 4                                                                 |
| 25 | the last two digits are 00, 25, 50 or 75                                                  |

For other divisors, split them into co-prime factors and test each: a number is divisible by 15 if it is divisible by 3 and by 5. For example, 364 is divisible by 7 because  $36 - 2 \times 4 = 28$ .

## HCF and LCM

### To find

### Formula or method

|                                                                        |                                                                   |
|------------------------------------------------------------------------|-------------------------------------------------------------------|
| HCF from prime factors                                                 | multiply the lowest power of each prime common to all the numbers |
| LCM from prime factors                                                 | multiply the highest power of every prime that appears            |
| Link between HCF and LCM (two numbers a and b only)                    | $\text{HCF} \times \text{LCM} = a \times b$                       |
| HCF of fractions in lowest terms                                       | HCF of numerators/LCM of denominators                             |
| LCM of fractions in lowest terms                                       | LCM of numerators/HCF of denominators                             |
| Largest number that divides a, b and c leaving the same remainder      | HCF of the positive differences $a - b$ , $b - c$ and $a - c$     |
| Largest number that divides a and b leaving remainders $r_1$ and $r_2$ | $\text{HCF}(a - r_1, b - r_2)$                                    |
| Smallest number above r leaving remainder r when divided by x, y and z | $\text{LCM}(x, y, z) + r$                                         |

| To find                                                                                            | Formula or method         |
|----------------------------------------------------------------------------------------------------|---------------------------|
| Smallest number leaving remainders $x - k$ , $y - k$ and $z - k$ when divided by $x$ , $y$ and $z$ | $\text{LCM}(x, y, z) - k$ |

For example, the smallest number above 3 that leaves remainder 3 when divided by 6, 8 and 12 is  $\text{LCM}(6, 8, 12) + 3 = 24 + 3 = 27$ .

## Remainders and last digits

- $\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$ , where the remainder is less than the divisor.
- The remainder of a sum or product equals the remainder of the sum or product of the individual remainders. For  $23 \times 17$  divided by 5, the remainders are 3 and 2, and  $3 \times 2 = 6$  leaves 1, so 391 leaves remainder 1.
- For a whole number  $a$  greater than 1,  $(a + 1)^n$  leaves remainder 1 when divided by  $a$ .
- For the same  $a$ ,  $(a - 1)^n$  divided by  $a$  leaves remainder 1 when  $n$  is even, and  $a - 1$  when  $n$  is odd.
- $a^n - b^n$  is divisible by  $a - b$  for every  $n$ , and by  $a + b$  when  $n$  is even.  $a^n + b^n$  is divisible by  $a + b$  when  $n$  is odd.
- Fermat's little theorem: if  $p$  is prime and  $a$  is not a multiple of  $p$ ,  $a^{(p - 1)}$  leaves remainder 1 when divided by  $p$ .

The last digit of a power depends only on the last digit of the base, and repeats in a cycle:

| Last digit of the base | Last digits of successive powers   |
|------------------------|------------------------------------|
| 0, 1, 5, 6             | always the same digit              |
| 2                      | 2, 4, 8, 6 (repeats every 4)       |
| 3                      | 3, 9, 7, 1 (repeats every 4)       |
| 4                      | 4, 6 (odd powers 4, even powers 6) |
| 7                      | 7, 9, 3, 1 (repeats every 4)       |
| 8                      | 8, 4, 2, 6 (repeats every 4)       |
| 9                      | 9, 1 (odd powers 9, even powers 1) |

For a cycle of 4, divide the exponent by 4 and use the remainder as the position in the cycle; a remainder of 0 means the fourth position.

## Factors, trailing zeros and series

- If  $N = p^a \times q^b \times r^c$  with  $p, q$  and  $r$  prime,  $N$  has  $(a + 1) \times (b + 1) \times (c + 1)$  factors.
- The sum of those factors is  $(p^{a+1} - 1)/(p - 1) \times (q^{b+1} - 1)/(q - 1) \times (r^{c+1} - 1)/(r - 1)$ .
- The number of zeros at the end of  $n!$  is the sum of the whole-number parts of  $n/5, n/25, n/125$  and so on. For  $100!$ , that is  $20 + 4 = 24$ .
- Sum of the first  $n$  natural numbers:  $n \times (n + 1)/2$ . Of their squares:  $n \times (n + 1) \times (2n + 1)/6$ . Of their cubes:  $(n \times (n + 1)/2)^2$ .
- Sum of the first  $n$  odd numbers:  $n^2$ . Of the first  $n$  even numbers:  $n \times (n + 1)$ .

**Worked example.** Find the remainder when  $3^{21}$  is divided by 5.

Powers of 3 end in 3, 9, 7, 1, in a cycle of 4. 21 divided by 4 leaves remainder 1, so  $3^{21}$  ends in 3, and any number ending in 3 leaves remainder 3 when divided by 5. Check with Fermat's little theorem:  $3^4$  leaves remainder 1 when divided by 5, so  $3^{20} = (3^4)^5$  also leaves 1, and  $3^{21} = 3^{20} \times 3$  leaves  $1 \times 3 = 3$ .

Squares up to 30, cubes up to 15 and the fraction table in the percentages section are worth knowing by heart. They turn many calculations into recognition, which saves time in every topic on this sheet.

## Before the test

A few checks catch a lot of avoidable mistakes:

- **Units.** Convert between km/h and m/s, months and years, and minutes and hours before substituting.
- **The base of a percentage.** Profit and loss are on cost price, discount is on marked price, and "A is x% more than B" uses B as the base.
- **Average speed.** Total distance over total time, never the average of the speeds.
- **Order.** Decide whether order matters before choosing between permutations and combinations.

- **"At least one"**. Usually fastest as  $1 - P(\text{none})$ .
- **Options**. Estimate the answer before calculating, and rule out options that are clearly too large or too small.
- **The final quantity**. Many wrong answers are correct calculations of a different quantity, such as compound interest instead of the amount, or the stream's speed instead of the boat's.